Homework 1 – Due: Friday, January 21, 2005

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1

Fix any finite set S and let the power set of S be denoted as $\mathcal{P}(S)$. Let R be the relation between elements of $\mathcal{P}(S)$ such that ARB if and only if there is a bijection between A and B.

- Show that R is an equivalence relation.
- Define a relation $R_1 \subset R$ that is reflexive and symmetric but not transitive.
- Define a relation $R_2 \subset R$ that is reflexive and transitive but not symmetric.
- Define a relation $R_3 \subset R$ that is symmetric and transitive but not reflexive.

2

Prove the following mathematical identity on integers:

$$\sum_{i=1}^{n} i^3 = \left(\sum_{i=1}^{n} i\right)^2$$

It may be useful to recall that $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$.

3

We proved in class the theorem stating that the tiling problem can always be solved. Based on the proof of that theorem, extract an algorithm that tiles a board. Implement your algorithm in a computer program (using the language of your choice). The program should ask the user to input m (the number of squares along a side of the board – a power of 2) and the coordinates (s_x, s_y) of the initial "special" square (where valid coordinates range between 0 and $m-1$). Your program should then print out a tiled board, using letters to distinguish the different tile pieces (you don't have to worry about ensuring too strictly that adjacent tiles are not represented using the same letter).

For instance, here is a sample run:

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Please enter
  m: 4
 sx: 1
 sy: 0

B # C C
B B A C
D A A E
D D E E
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